LITERATURE CITED

1. N. V. Karlov and A. M. Prokhorov, Usp. Fiz. Nauk, 123, No. 1 (1977).
2. V. S. Letokhov, Usp. Fiz. Nauk, 125, No. 1 (1978).
3. V. I. Gol'danskii, V. A. Namiot, and R. V. Khokhlov, Zh. Eksp. Teor. Fiz., 70, No. 6 (1976).
4. N. G. Basov, É. M. Belenov, V. A. Isakov, Yu. S. Leonov, E. P. Markin, A. N. Oraevskii, V. I. Romanenko, and N. B. Ferapontov, Pis'ma Zh. Eksp. Teor. Fiz., 22, No. 4 (1975).
5. Yu. V. Brzhazovskii, Yu. S. Kusner, A. K. Rebrov, B. I. Troshin, and V. P. Chebotaev, Pis'ma Zh. Eksp. Teor. Fiz., 23, No. 5 (1976).
6. V. V. Levdanskii, Inzh.-Fiz. Zh., 31, No. 1 (1976).
7. G. W. Sears, J. Chem. Phys., 22, No. 7 (1954).
8. P. Clausing, Ann. Phys., 12, 961 (1932).
9. A. V. Luikov, T. L. Perelman, V. V. Levdansky, V. G. Leitsina, and N. V. Pavlyukevich, Int. J. Heat Mass Transfer, 17, No. 9 (1974).
10. A. I. Ivanovskii, O. K. Kostko, and A. V. Fedynskii, Tr. TsAO, No. 46 (1963).
11. E. Ya. Kogan and V. N. Mal'nev, Zh. Eksp. Teor. Fiz., 75, No. 3(9) (1978).

EFFECT OF A COOLANT TEMPERATURE JUMP ON A CLAD FUEL ROD

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An analytic expression is derived for the time dependence of the temperature distribution in cylindrical clad fuel elements. The transient state results from a jump in the coolant temperature.

We assume that the coolant temperature in a nuclear reactor operating in a steady state is instantaneously increased from T_F to $T_F + \Delta T$. Such an increase can arise because of leakage of core coolant, as the result of an accident, or for other reasons. It is assumed that the heat release (in fission) rate remains unchanged, and consequently the temperature of the fuel and cladding increases until a new steady state is reached. The solution of the transient problem resulting from such a hypothetical accident is important for two reasons. First, it is expedient and necessary to know a priori whether the temperature of the fuel or cladding at the end of the transient process reaches dangerous values which imperil the effectiveness and safety of the operation of the facility [1]. Second, it is important to establish from the variation of temperature with time whether the material, which was already subjected to a heat load because of the spatial temperature gradient, experiences further heat loads as a consequence of the accidental jump in temperature. Heat loads are particularly dangerous in a transient process.

Thus, the present problem is reduced to an emergency situation which can occur in a nuclear reactor, and its solution would permit an appropriate choice of materials and operating conditions of the facility. We assume that the fuel element and cladding are homogeneous and isotropic and have constant physical characteristics. In cylindrical coordinates the heat-conduction equations for a fuel element and cladding are, respectively,

$$\frac{\alpha_1}{\alpha_2}\left[\frac{\partial^2\tilde{\theta}_1}{\partial\xi^2}+\frac{1}{\xi}\frac{\partial\tilde{\theta}_1}{\partial\xi}+\frac{\partial^2\tilde{\theta}_1}{\partial\zeta^2}+q_\theta\cos\pi b\zeta\right]=\frac{\partial\tilde{\theta}_1}{\partial \mathrm{Fo}}, \tag{1}$$

$$\frac{\partial^2\tilde{\theta}_2}{\partial\xi^2}+\frac{1}{\xi}\frac{\partial\tilde{\theta}_2}{\partial\xi}+\frac{\partial^2\tilde{\theta}_2}{\partial\zeta^2}=\frac{\partial\tilde{\theta}_2}{\partial \mathrm{Fo}}. \tag{2}$$

The functions $\tilde{\theta}_1$ and $\tilde{\theta}_2$ and the parameters in Eqs. (1) and (2) are defined as follows:

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$$\tilde{\theta}_1 = T_f - (T_F + \Delta T),\ \tilde{\theta}_2 = T_c - (T_F + \Delta T),\ \mathrm{Fo} = \alpha_2 \frac{t}{R_1^2},$$
$$\xi = r/R_1,\ \zeta = z/R_1,\ q_\theta = q_0 R_1^2/K_f, \tag{3}$$
$$\mathrm{Bi} = hR_1/K_c,\ b = R_1/2(L+\delta),\ H_0 = K_f/K_c.$$

The origin of the cylindrical coordinate system is located at the geometric center of the rod, and the ζ axis coincides with the axis of the rod.

According to [2, 3] the boundary and initial conditions of the problem have the form

$$\partial\tilde{\theta}_1/\partial\xi = 0 \quad \text{at} \quad \xi = 0, \tag{4}$$

$$\tilde{\theta}_1 = 0 \quad \text{at} \quad \zeta = \pm\zeta_{\mathrm{extr}} = \frac{L+\delta}{R_1}, \tag{5}$$

$$\partial\tilde{\theta}_1/\partial\zeta \quad \text{at} \quad \zeta = 0, \tag{6}$$

$$\frac{\partial\tilde{\theta}_2}{\partial\xi} + \mathrm{Bi}\,\tilde{\theta}_2 = 0 \quad \text{at} \quad \xi = \xi_0 = R_2/R_1, \tag{7}$$

$$\tilde{\theta}_2 = 0 \quad \text{at} \quad \zeta = \pm\zeta_{\mathrm{extr}}, \tag{8}$$

$$\partial\tilde{\theta}_2/\partial\zeta = 0 \quad \text{at} \quad \zeta = 0, \tag{9}$$

$$\tilde{\theta}_1 = \tilde{\theta}_2 \quad \text{at} \quad \xi = 1, \tag{10}$$

$$H_0 \frac{\partial\tilde{\theta}_1}{\partial\xi} = \frac{\partial\tilde{\theta}_2}{\partial\xi} \quad \text{at} \quad \xi = 1, \tag{11}$$

$$\tilde{\theta}_1(\xi,\ \zeta,\ \mathrm{Fo}) = \tilde{\theta}_1(\xi,\ \zeta,\ 0),\ \mathrm{Fo} = 0, \tag{12}$$

$$\tilde{\theta}_2(\xi,\ \zeta,\ \mathrm{Fo}) = \tilde{\theta}_2(\xi,\ \zeta,\ 0),\ \mathrm{Fo} = 0. \tag{13}$$

The functions $\tilde{\theta}_1(\xi, \zeta, 0)$ and $\tilde{\theta}_2(\xi, \zeta, 0)$ for a steady-state problem are easy to find by the method used in [2, 3]:

$$\tilde{\theta}_1(\xi,\ \zeta,\ 0) = \cos\beta_0\zeta\left[AI_0(\beta_0\xi) + \frac{q_\theta}{\pi^2 b^2}\right] - \Delta T, \tag{14}$$

$$\tilde{\theta}_2(\xi,\ \zeta,\ 0) = D\cos\beta_0\zeta\,[CI_0(\beta_0\xi) + K_0(\beta_0\xi)] - \Delta T. \tag{15}$$

The constants β_0, C, D, and A are given by Eqs. (27)-(30). The solution of Eqs. (1) and (2) is given by the sum of two functions

$$\tilde{\theta}_1 = \theta_1 + \theta_1^*, \tag{16}$$

$$\tilde{\theta}_2 = \theta_2 + \theta_2^*, \tag{17}$$

where θ_1 and θ_2 are solutions of Eqs. (1) and (2) for the steady state ($T_F + \Delta T$ is the coolant temperature); θ_1^* and θ_2^* are solutions of the homogeneous differential equations corresponding to Eqs. (1) and (2). The uniqueness of the solutions $\tilde{\theta}_1$ and $\tilde{\theta}_2$ is proved in [4].

The functions θ_1 and θ_2 describe the temperature distribution in the rod at the end of the transient process

$$\theta_1(\xi,\ \zeta) = \tilde{\theta}_1(\xi,\ \zeta,\ \infty) \text{ and } \theta_2(\xi,\ \zeta) = \tilde{\theta}_2(\xi,\ \zeta,\ \infty)$$

and satisfy the same boundary conditions as the functions $\tilde{\theta}_1$ and $\tilde{\theta}_2$; their analytic expressions are given in [2] and [3]. They differ from the functions $\tilde{\theta}_1(\xi, \zeta, 0)$ and $\tilde{\theta}_2(\xi, \zeta, 0)$ only in the presence of ΔT.

Let us now determine the functions θ_1^* and θ_2^*, which are the solutions of the following system of homogeneous differential equations:

$$\frac{\alpha_1}{\alpha_2}\left[\frac{\partial^2\theta_1^*}{\partial\xi^2} + \frac{1}{\xi}\frac{\partial\theta_1^*}{\partial\xi} + \frac{\partial^2\theta_1^*}{\partial\zeta^2}\right] = \frac{\partial\theta_1^*}{\partial\,\mathrm{Fo}}, \tag{18}$$

$$\frac{\partial^2\theta_2^*}{\partial\xi^2} + \frac{1}{\xi}\frac{\partial\theta_2^*}{\partial\xi} + \frac{\partial^2\theta_2^*}{\partial\zeta^2} = \frac{\partial\theta_2^*}{\partial\,\mathrm{Fo}}. \tag{19}$$

Identical boundary conditions are imposed on the functions θ_1^*, θ_2^* and θ_1, θ_2. The initial conditions for θ_1^* and θ_2^* have the form

$$\theta_1^* = -\Delta T \text{ for } \mathrm{Fo} = 0, \tag{20}$$

$$\theta_2^* = -\Delta T \text{ for } \mathrm{Fo} = 0. \tag{21}$$

It is clear that the solutions θ_1^* and θ_2^* do not depend on the source q_θ.

The solutions of Eqs. (18) and (19) can be found by the method of separation of variables. As a result, we have

$$\theta_1^* = \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} A_{mn}\cos\beta_n\zeta J_0(\mu_m\xi)\exp[-(\mu_m^2+\beta_n^2)\,\mathrm{Fo}], \tag{22}$$

$$\theta_2^* = \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} B_{mn}\cos\beta_n\zeta\,[C_mJ_0(\mu_m\xi)+Y_0(\mu_m\xi)]\exp[-(\mu_m^2+\beta_n^2)\,\mathrm{Fo}]. \tag{23}$$

We determine the constants C_m and μ_m from Eqs. (7), (10), and (11), and β_n from Eqs. (5) and (8). The determination of the constants A_{mn} and B_{mn} requires using (10), (11), (20), and (21), the orthogonality and completeness properties of a set of trigonometric functions [5] in the domain $|-\zeta_{extr}, \zeta_{extr}|$, and orthogonality of an appropriate linear combination of Bessel functions [4]. The complete solution has the form

$$\tilde{\theta}_1 = \left[AI_0(\beta_0\xi)+\frac{q_\theta}{\pi^2b^2}\right]\cos\beta_0\zeta + \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} A_{mn}\cos\beta_n\zeta J_0(\mu_m\xi)\exp[-(\mu_m^2+\beta_n^2)\,\mathrm{Fo}], \tag{24}$$

$$\tilde{\theta}_2 = D\cos\beta_0\zeta CI_0(\beta_0\xi)+K_0(\beta_0\xi)+\sum_{n=0}^{\infty}\sum_{m=1}^{\infty} B\cos\beta_n\zeta C_mJ_0(\mu_m\xi)+Y_0(\mu_m\xi)\exp[-(\mu_m^2+\beta_n^2)\,\mathrm{Fo}]. \tag{25}$$

The eigenvalues μ_m are solutions of the equation

$$\left[\left(\frac{1}{H_0}-1\right)J_1(\mu)J_0(\mu)\,\mathrm{Bi}\,Y_0(\mu\xi_0)-\mu Y_1(\mu\xi_0)+\mathrm{Bi}\,J_0(\mu\xi_0)-\mu J_1(\mu\xi_0)J_1(\mu)Y_0(\mu)\right.$$
$$\left.-\frac{J_0(\mu)Y_1(\mu)}{H_0}\right]\Big/[\mathrm{Bi}\,J_0(\mu\xi_0)-\mu J_1(\mu\xi_0)]=0. \tag{26}$$

The values of the coefficients in Eqs. (24) and (25) are:

$$\beta_n = (2n+1)\frac{\pi}{2\zeta_{extr}} \text{ for } n=0,\ 1,\ 2,\ 3,\ \ldots, \tag{27}$$

$$C = \frac{\beta_0K_1(\beta_0\xi_0)-\mathrm{Bi}\,K_0(\beta_0\xi_0)}{\beta_0I_1(\beta_0\xi_0)+\mathrm{Bi}\,I_0(\beta_0\xi_0)}, \tag{28}$$

$$D = \frac{AI_1(\beta_0)H_0}{CI_1(\beta_0)-K_1(\beta_0)}, \tag{29}$$

$$A = \frac{q_\theta}{(\pi b)^2}\,\frac{CI_1(\beta_0)-K_1(\beta_0)}{[CI_0(\beta_0)+K_0(\beta_0)]H_0I_1(\beta_0)-I_0(\beta_0)[CI_1(\beta_0)-K_1(\beta_0)]}, \tag{30}$$

$$C_m = \frac{\mu_mY_1(\mu_m\xi_0)-\mathrm{Bi}\,Y_0(\mu_m\xi_0)}{\mathrm{Bi}\,J_0(\mu_m\xi_0)-\mu_mJ_1(\mu_m\xi_0)}, \tag{31}$$

$$G_m = \frac{C_mJ_0(\mu_m)+Y_0(\mu_m)}{J_0(\mu_m)}, \tag{32}$$

$$L_m = \frac{1}{2}\left\{K_f[J_1^2(\mu_m)+J_0^2(\mu_m)]+\frac{K_c}{G_m^2}\xi_0^2[(C_mJ_1(\mu_m\xi_0)+Y_1(\mu_m\xi_0))^2+(C_mJ_0(\mu_m\xi_0)+Y_0(\mu_m\xi_0))^2]-\right.$$
$$\left.-\frac{K_c}{G_m^2}[(C_mJ_1(\mu_m)+Y_1(\mu_m))^2+(C_mJ_0(\mu_m)+Y_0(\mu_m))^2]\right\}, \tag{33}$$

$$A_{mn} = -\frac{2\Delta T(-1)^n}{\mu_m\beta_n\zeta_{extr}L_m}\left\{\left(K_f-K_c\frac{C_m}{G_m}\right)J_1(\mu_m)+K_c\frac{C_m}{G_m}\xi_0J_1(\mu_m\xi_0)+\frac{K_c}{G_m}[\xi_0Y_1(\mu_m\xi_0)-Y_1(\mu_m)]\right\}.$$

To describe and analyze the results obtained we present temperature profiles in a cylindrical clad fuel rod (Figs. 1-4). The profiles were calculated by using Eqs. (24) and (25) with the following values of the parameters: Bi = 1, 5; H_0 = 0.2, 0.5; α_2 = 0.068, 0.135, 0.270 cm²/sec for q_θ = 600°C, ζ_0 = 100, b = 0.0045, ξ_0 = 1.1, ΔT = 60°C.

For greater clarity the time in seconds rather than the Fo number was chosen as the independent variable. The time in terms of the Fo number is given by the relation Fo = 0.541t for H_0 = 0.2 (Figs. 1, 2, 4),

$Fo = 0.216t$ for $H_0 = 0.5$ (Fig. 2b), $Fo = 4\alpha_2 t$ (Fig. 3).

It follows from Fig. 1a, b that the values of the steady-state temperature (at $t = 0$ and $t = \infty$) increase with a decrease in the Biot number. In addition, for large values of the Biot number the temperature reaches a steady state more quickly ($t = \infty$). Physically this can be explained by the fact that if the removal of heat from the wall is equal to the inflow of heat to the wall, then with an increase in the heat-transfer coefficient (i.e., with an increase in the Biot number) the temperature difference between the cladding and coolant is decreased, and consequently the cladding temperature is decreased.

As a result the temperature of the fuel element reaches lower values. Similarly, for a decrease in the thermal conductivity of the cladding (i.e., with an increase in the Biot number) when the linear power of the fuel element, the external temperature of the cladding, and the heat-transfer coefficient remain constant, the rate of rise of temperature between the end surfaces of the cladding increases. Consequently, as the temperature at the inner surface of the cladding increases, the whole temperature distribution increases. Therefore, it is obvious that the Biot number is a measure of heat-transfer effectiveness.

Thus, the higher the Biot number the lower the temperature in the fuel rod, and the more rapidly the transient state is ended. It is found also that the temperature increases very much more rapidly at points farther from the fuel rod axis. This results from the fact that the temperature of the coolant is increased suddenly. The perturbation of the coolant temperature is propagated toward the center of the fuel element with a finite velocity.

Variations of temperature with time are shown in Fig. 2a. It is clear from the figure that for $Bi = 5$ the transient state at the outer surface of the cladding ($\xi = 1.1$) lasts 0.5 sec, and the temperature at the rod axis ($\xi = 0$) reaches an asymptotic value corresponding to the steady state in 2 sec. Figure 2a also shows the variation of temperature with time at a certain point of the cladding in contact with the coolant ($\xi = 1.1$) for two different values of the Biot number. Thus, we see that the higher the Biot number the lower the steady-state temperature, but the shorter the transient state. From this there follows the very interesting conclusion that it is desirable that the Biot number have a high value in a nuclear reactor, since in this case the materials will "work" at lower temperatures. However, in an emergency (jump in coolant temperature) when the transient state is shorter, high thermal stresses develop in the material. It is clear that the cladding experiences a higher stress than the fuel element itself, since the temperature in it rises more rapidly.

Thus, it is necessary to choose materials and system parameters so as to produce a compromise in which the steady-state temperature is appreciably lower than the critical temperature, and random thermal stresses do not impair the mechanical properties of the materials.

Figure 2b shows the time dependence of the temperature at a certain point on the rod axis ($\xi = 0$) and at

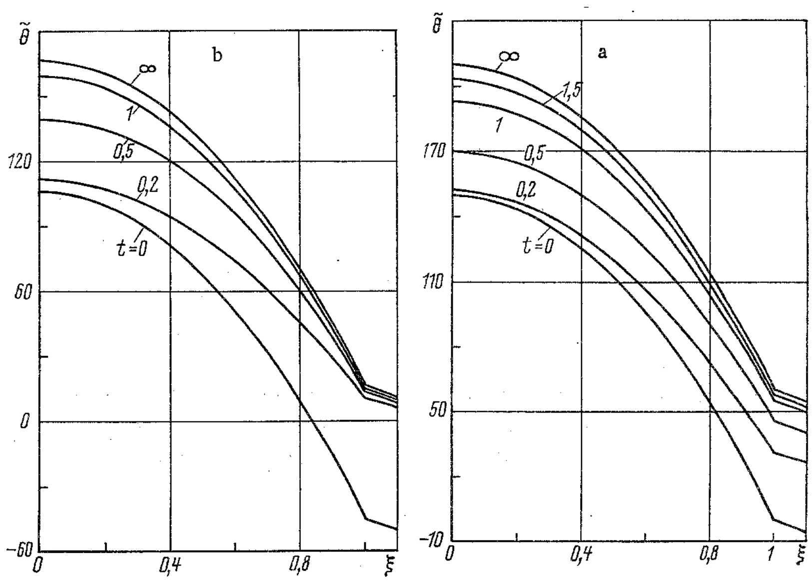

Fig. 1. Temperature $\tilde{\theta}$ as a function of the dimensionless radial coordinate ξ at several values of the time for Bi equal to a) 1 and b) 5; $H_0 = 0.2$, $\zeta = 0$.

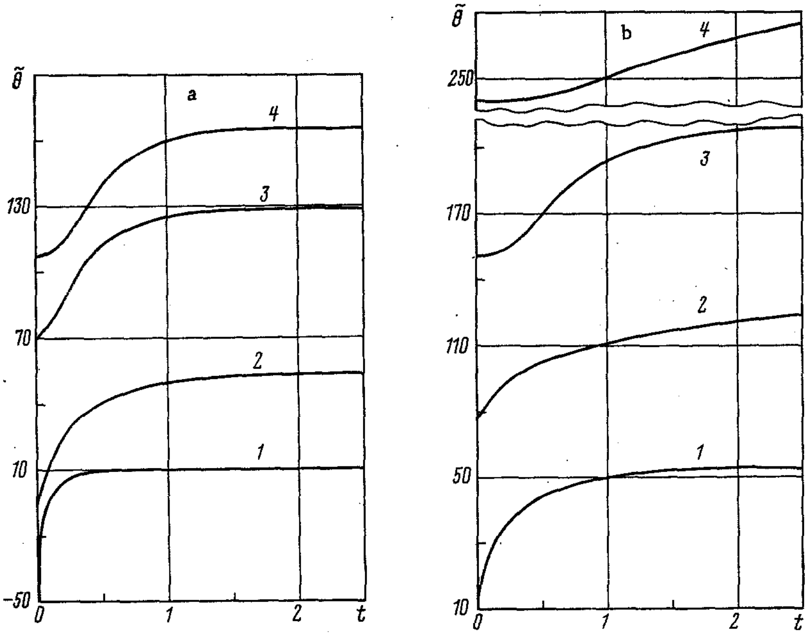

Fig. 2. Time dependence of $\tilde{\theta}$: a) for several points in the middle plane and for two different values of the Biot number Bi (H_0 = 0.2, ζ = 0): 1) Bi = 5, ζ = 1.1; 2) 1, 1.1: 3) 5, 0.5; 4) 5, 0; b) for two points in the middle plane and for two different values of the parameter H_0 (Bi = 1, ζ = 0): 1) H_0 = 0.2, ζ = 1.1; 2) 0.5, 1.1; 3) 0.2, 0; 4) 0.5, 0.

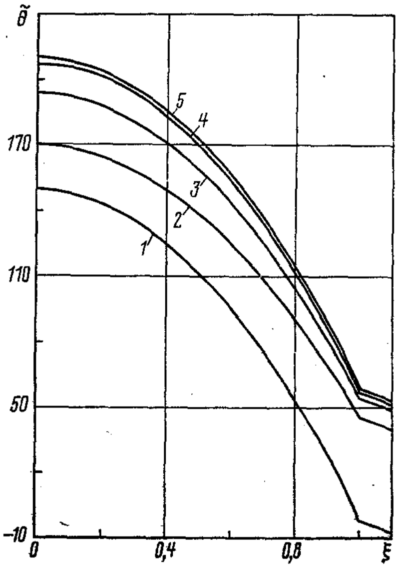

Fig. 3

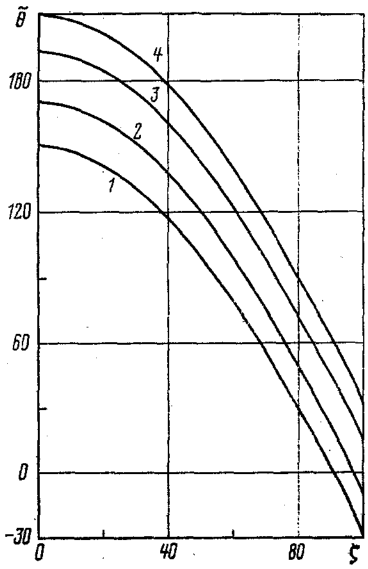

Fig. 4

Fig. 3. Temperature $\tilde{\theta}$ as a function of the dimensionless radial coordinate ξ for several values of the time and three different values of the thermal diffusivity α_2 of the cladding (Bi = 1, H_0 = 0.2, ζ = 0: 1) t = 0; 2) t = 1, α_2 = 0.068; 3) 1, 0.135; 4) 1, 0.27; 5) t = ∞.

Fig. 4. Temperature $\tilde{\theta}$ as a function of the dimensionless axial coordinate ζ for several values of the time (Bi = 1, H_0 = 0.2, ξ = 0) : 1) t = 0; 2) 0.5; 3) 1; 4) t = ∞.

a certain point on the outer surface of the cladding ($\xi = 1.1$) for two different ratios H_0 of the thermal conductivity of the rod to that of the cladding. It might seem that the Biot number and the ratio H_0 would affect the temperature distribution differently. Actually the steady-state temperature increases with an increase in H_0, whereas the transient state is prolonged. Thus, it is necessary to choose materials which can withstand thermal loads under steady-state conditions and mechanical loads during the transient state.

Figure 3 shows the radial temperature distribution before the perturbation ($t = 0$), 1 sec after the perturbation occurred, and at the end of the transient ($t = \infty$) for several values of the thermal diffusivity α_2 of the cladding. As was shown, the thermal diffusivity does not affect the steady-state temperature distribution ($t = 0$ and $t = \infty$), but only the transient distribution. It is quite clear that the rate of rise of temperature increases with increasing α_2. Therefore, it is natural to take a rather small value of α_2 in order to avoid excessive thermal overloading.

It can definitely be stated that in problems of the type considered, the choice of Biot number and the ratio H_0 should be based on a compromise when possible without taking too high a value of the thermal diffusivity α_2 (taking account of requirements imposed on other units of the facility).

Figure 4 shows that the axial temperature curves at various times are congruent with the steady-state curve ($t = 0$ and $t = \infty$). This can be understood physically from the fact that the perturbation actually reaches all points on a line parallel to the axis with the same delay time.

The present work is a development and completion of research started in [6].

The numerical calculations were performed on a Cyber-76 computer at the Interuniversity Computation Center of Northeast Italy with the permission of the CNR.

NOTATION

r, z, radial and axial coordinates: R_1, R_2, radii of fueled region and rod; L, half-height of rod; δ, extrapolation distance; q_0, heat release at center of rod; h, heat-transfer coefficient; K_f, K_c, thermal conductivities of fuel and cladding; α_1, α_2, thermal diffusivities of fuel and cladding; T_f, T_c, temperatures of fuel and cladding; T_F, coolant temperature before jump; ΔT, jump in coolant temperature; $\tilde{\theta}_1$, $\tilde{\theta}_2$, temperatures of fuel and cladding according to Eq. (3); θ_1^*, θ_2^*, temperatures of fuel and cladding according to Eqs. (16) and (17); t, time; J_0, J_1, zeroth- and first-order Bessel functions of the first kind; Y_0, Y_1, zeroth- and first-order Bessel functions of the second kind; I_0, I_1, zeroth- and first-order modified Bessel functions of the first kind; K_0, K_1, zeroth- and first-order modified Bessel functions of the second kind; Bi, Biot number (3); Fo, Fourier number (3); ζ_{extr}, extrapolated height according to Eq. (5); μ, eigenvalue, Eq. (26); β, eigenvalue, Eq. (27).

LITERATURE CITED

1. M. M. El-Wakil, Nuclear Heat Transport, International Textbook Co., Scranton (1971)
2. E. Lorenzini and M. Spiga, "Distribuzione di temperatura in una barretta di combustibile nucleare con gap e cladding, con temperatura del refrigerante variabile," Tecnica Italiana, in press.
3. E. Lorenzini, "Profili di temperatura in barre di combustibile nucleare con cladding, Ingegneria, Nos. 7-8, July-August (1975).
4. M. Spiga, Graduation Thesis, Univ. of Bologna, Dept. of Engng. (1975-76).
5. C. R. Wylie, Advanced Engineering Mathematics, McGraw-Hill, New York (1966).
6. E. Lorenzini and M. Spiga, "Effetti di una improvvisa variazione di temperatura del refrigerante in una barra di combustibile nucleare con cladding," Ingegneria, in press.